CONTROL OF A THREE-PHASE INVERTER, WITH A DELTA-WYE TRANSFORMER, USING THE PARK TRANSFORMATION

José Augusto da Matta Guedes and Ivo Barbi
Instituto de Eletrônica de Potência – INEP
Departamento de Engenharia Elétrica
Universidade Federal de Santa Catarina – UFSC
Caixa Postal: 5119 – CEP: 88040-970
Florianópolis – Santa Catarina – Brasil
Internet: www.inep.ufsc.br – E-mail: jose@inep.ufsc.br and ivobarbi@inep.ufsc.br

***Abstract* – This article has the goal of presenting the equations of a sinusoidal three-phase inverter, which uses an isolating transformer, with its primary side connected in delta and its secondary side in a four wire wye (three-phase with a ground wire), and a passive LC filter on the primary side and supplies balanced and unbalanced non-linear loads (in this case, full bridge rectifiers with capacitive filters).**

The dynamic model of the inverter, transformer, filter and loads in dq0 coordinates is presented. Since the resulting dq inputs of the plant are coupled, a decoupling transformation is proposed. The numerical simulation results validate the theoretical analysis, showing the good performance of the system in steady-state when using the proposed controller with both balanced and unbalanced non-linear loads.

***Keywords* – Three-Phase Inverters, unbalanced load, uninterruptible power supply.**

I. INTRODUCTION

Nowadays, a considerable advance in power electronics is observed. The development of new techniques stimulate innovative ideas that motivate the study of new structures, new techniques of modulation, modeling and control.

An increasing in the consumption of energy in the diverse segments of world-wide electric market is observed, in its majority for the great demand of electronic loads, or nonlinear loads, as for example: television sets, videos, computers, fax, converters for telecommunications, electronic ballast, and others.

Inverters or DC-AC converters present a great number of applications, as for example in renewed energy systems and drive of electric machines, but mainly in uninterrupted sources of energy (Uninterruptible Power Supply UPS or No-break).

UPS Systems can supply power in emergency situations for critical loads. The least energy lack, even during fractions of second, can cause losses of information or affect very important processes in the most diverse areas: systems of communication, industrial data processing centers, computers in airlines, airports, or in the units of intensive treatment in hospitals.

Some characteristics contribute for the distortions in the output inverter voltages, such as: high crest factor in load currents, caused by not controlled diodes rectifiers with capacitive filter (inherent non linearity of the PWM inverter), quality of the DC link bar, drop voltage on the semiconductors, impedance of the transformer used for galvanic insulation and unbalanced non linear loads.

As the DC-AC converter is the most important device in a UPS system, is desirable that it will have the highest quality as it is possible.

The structure presented in Figure 1 is one of the possible configurations. This topology is distinguished for a three-phase inverter configuration, which is one of the most adequate to be used. The reasons could be set as: galvanic insulation from the load; changeable output voltage and frequency; the star connection in the secondary allows the use of the neutral conductor; the delta connection in the primary permits the use of the line the three-phase inverter voltage.

Prospecting research were done in this way, developing topologies, LC series filtering, commutation analysis, control strategies, modeling and modulation techniques [5], [6], [7], [8], [9] and [10].In [5], a new real-time digital compensator implemented in a digital-signal-processor (DSP)-based system that adapts to both nonlinear and unbalanced load without exact knowledge of the output-filter components. The compensation is based on the vectorial voltage error on the output.

In [6], a three-layer control scheme is proposed. It consist of a proportional compensator in stationary $\alpha\beta$ frame, an integral controller in synchronous frame to compensate the fundamental component and a selective harmonic compensator in stationary frame based in a pass-band FIR filter with unity gain and zero phase at the selective harmonics. The delay introduced by the plant and the inner current control loop is compensated by introduce a block delay with positive feedback in the harmonic control layer.

In [7], selected harmonics on the output are identified by a discrete Fourier transformation (DFT). The mean values of selected harmonics of the three phases are used as feedback to an integrating controller, which is applied to the control signal of the inverter. In this way, selected balanced can be removed

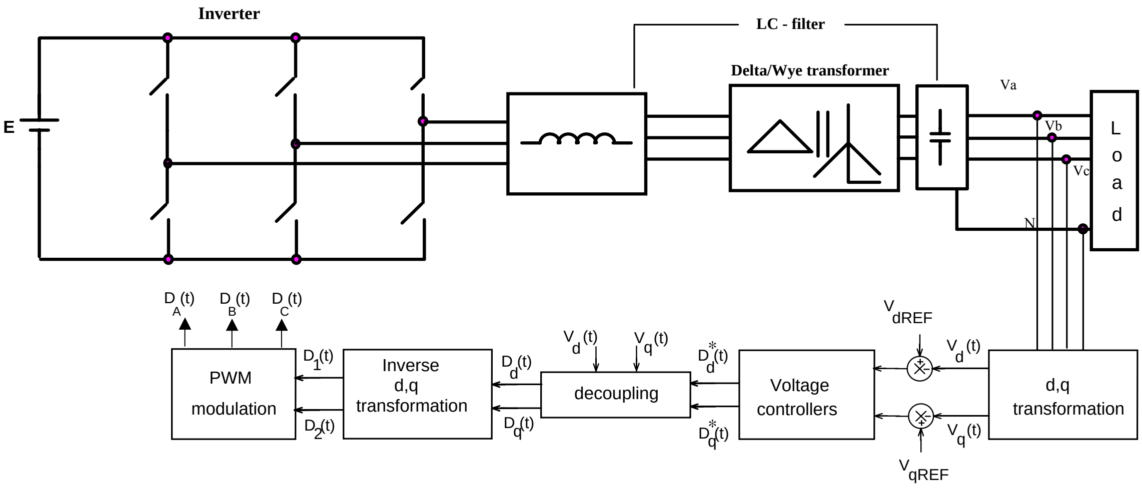

Fig. 1 – Structural diagram and the control employed harmonics

Unfortunately, umbalanced distortion cannot be removed by this method.

In [8], a repetitive learning controller is used instead to adapt the switching function to compensate for the harmonics. This scheme require also very good information about the filter components in order to apply the inverse filter transfer function.

In [9], a new digital voltage controller based on the internal model principle for three-phase inverters with ΔY transformer at the output is proposed. The proposed digital controller is not prone to produce DC components, which that can lead to the output transformer saturation.

In [10], a deadbeat controller with voltage and current feedback is applied. The control scheme uses the state feedback information to compensate for the inductor voltage drop. This scheme is very dependent on precise knowledge of the filter components and requires measuring of both filter current, load current, and output voltage. The scheme also includes prediction algorithms to compensate for the controller and system delays. Unfortunately, the scheme will not compensate for the inverter-induced harmonics.

Among the presented proposals, solutions do not exist that can be considered standards. These options are many times discarded for the great complexity and high cost. In this direction this research is intended to fill this gap through a simple control and low cost.

In this context it is presented the study of the Park transformation applied to the inverter, aiming to generate viable alternatives to resolve some of the problems found in the conventional inverting structures of form to get an inverter of high quality, high efficiency, high density of power, low audible noise, low cost of implementation and eventually sinusoidal voltage on the load with the fewer harmonic content, for linear or nonlinear loads. The controller used is a PID, and the control technique is the instantaneous average values. The presented results of numerical simulation validate the theoretical development, demonstrating the good performance of the system in stead state with the considered controller, as much with unbalanced not linear load as well as balanced one. Figure 1 presents the structural diagram as well as the control system used.

In section 2, the concepts of the output filter are shown. Section 3 presents the modeling of the inverter. In section 4 the decoupling concepts used here are presented.

In section 5 the simulation results are presented and in section 6 conclusions are drawn and the results are analyzed.

II. OUTPUT FILTER MODELING

The high frequency operation of the switches produces undesirable interference (harmonics) at the output of the inverter. For this reason, it is necessary to place, at the output of the primary side of the transformer, a filter to eliminate this high frequency and allow only the fundamental component of the voltage to pass. This filter is an LC filter and is presented in Fig. 2.

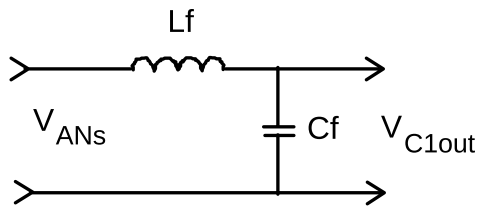

Fig. 2 – Output LC filter.

Thus, an equivalent circuit for the output voltages is obtained. The desired output voltages will be represented by V_{C1out}, V_{C2out} and V_{C3out}.

The transfer function of the circuit of Fig. 2, for the three outputs, is given in matrix form by:

$$\left\{ \begin{array}{l} \dfrac{d^2}{dt^2}\begin{bmatrix} V_{C1out}(t) \\ V_{C2out}(t) \\ V_{C3out}(t) \end{bmatrix} + \dfrac{1}{R\cdot Cf}\cdot\dfrac{d}{dt}\begin{bmatrix} V_{C1out}(t) \\ V_{C2out}(t) \\ V_{C3out}(t) \end{bmatrix} + \\ \dfrac{1}{Lf\cdot Cf}\cdot\begin{bmatrix} V_{C1out}(t) \\ V_{C2out}(t) \\ V_{C3out}(t) \end{bmatrix} = \dfrac{1}{Lf\cdot Cf}\cdot\begin{bmatrix} V_{AB}(t) \\ V_{BC}(t) \\ V_{CA}(t) \end{bmatrix} \end{array} \right. \tag{1}$$

Where:

V_{Cout} - Voltage on the filter capacitor

Lf - filter inductance.
R - Resistance.
$V_{AB}(t)$ - Line voltage of the inverter.

III. MODELING THE INVERTER

The three-phase line voltages of the inverter, as a function of the input voltage and the duty cycles of the switches of the three-phase inverter, are expressed, in matrix form, by equation (2).

$$\begin{bmatrix} V_{AB}(t) \\ V_{BC}(t) \\ V_{CA}(t) \end{bmatrix} = E \cdot \begin{bmatrix} D_{AB}(t) \\ D_{BC}(t) \\ D_{CA}(t) \end{bmatrix} \quad (2)$$

Where:

E - Continuous input voltage.
$D_{AB}(t)$ - Duty Cicle.

Therefore, from the secondary side of the transformer connected in wye and with the primary side in delta and considering still, for the sake of simplification, that the coupling, k, between the windings of a common leg of the core is equal to 1 and that those of adjacent legs are equal to 0.5, the equation, in matrix form, can be written as:

$$\begin{bmatrix} V_{ANs}(t) \\ V_{BNs}(t) \\ V_{CNs}(t) \end{bmatrix} = \frac{\sqrt{L_{pri} \cdot L_{\sec}}}{L_{pri}} \cdot \begin{bmatrix} V_{C1out}(t) \\ V_{C2out}(t) \\ V_{C3out}(t) \end{bmatrix} \quad (3)$$

Where:

$V_{ANs}(t)$ - Secondary voltage of the transformer.
L_{pri} - Self inductance of the primary.
L_{sec} - Self inductance of the secundary.

Substituting equations (2) and (3) into (1), we have:

$$\frac{d^2}{dt^2}\begin{bmatrix} V_{ANs}(t) \\ V_{BNs}(t) \\ V_{CNs}(t) \end{bmatrix} + \frac{1}{R \cdot Cf} \cdot \frac{d}{dt}\begin{bmatrix} V_{ANs}(t) \\ V_{BNs}(t) \\ V_{CNs}(t) \end{bmatrix} + \frac{1}{Lf \cdot Cf} \cdot \begin{bmatrix} V_{ANs}(t) \\ V_{BNs}(t) \\ V_{CNs}(t) \end{bmatrix} = \frac{1}{Lf \cdot Cf} \cdot E \cdot \frac{\sqrt{L_{pri} \cdot L_{\sec}}}{L_{pri}} \cdot \begin{bmatrix} D_{AB}(t) \\ D_{BC}(t) \\ D_{CA}(t) \end{bmatrix} \quad (4)$$

Defining the following vectors:

$$\overrightarrow{D_{ABC}} = \begin{bmatrix} D_{AB}(t) \\ D_{BC}(t) \\ D_{CA}(t) \end{bmatrix}, \overrightarrow{V_{ABCNs}} = \begin{bmatrix} V_{ANs}(t) \\ V_{BNs}(t) \\ V_{CNs}(t) \end{bmatrix} \quad (5)$$

Substituting equation (5) in (4), we have:

$$\frac{d^2}{dt^2}\overrightarrow{V_{ABCNs}} + \frac{1}{R \cdot Cf} \cdot \frac{d}{dt}\overrightarrow{V_{ABCNs}} + \frac{1}{Lf \cdot Cf} \cdot \overrightarrow{V_{ABCNs}} = \frac{1}{Lf \cdot Cf} \cdot E \cdot \frac{\sqrt{L_{pri} \cdot L_{\sec}}}{L_{pri}} \cdot \overrightarrow{D_{ABC}} \quad (6)$$

So, from **[1]**, it is known that the Park Transformation applied to any $\overline{X_{ABC}}$ vector is defined by:

$$\overline{X_{dq0}} = \overline{B}^{-1} \cdot \overline{X_{ABC}} \text{ or } \overline{X_{ABC}} = \overline{B} \cdot \overline{X_{dq0}} \quad (7)$$

Where $\overline{X_{ABC}}$ represents the vector of variables before the transformation, $\overline{B}^{-1}$ represents the matrix that performs the transformation and $\overline{X_{dq0}}$ represents the vector of variables that result from the Park Transformation applied to $\overline{X_{ABC}}$. Therefore, matrix $\overline{B}^{-1}$ is defined by:

$$\overline{B}^{-1} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ sin(\omega \cdot t) & sin(\omega \cdot t - 120°) & sin(\omega \cdot t + 120°) \\ \cos(\omega \cdot t) & \cos(\omega \cdot t - 120°) & \cos(\omega \cdot t + 120°) \end{bmatrix} \quad (8)$$

We also know from **[1]** that, in order to secure invariant power, the transformation must be orthogonal. In other words:

$$\overline{B}^{-1} = \overline{B}^{t} \text{ or } \overline{B} = \overline{B}^{-1t} \quad (9)$$

Thus, the equations that describe the system in dq0 coordinates are expressed as:

$$\begin{cases} \frac{d^2V_d(t)}{dt^2} + \frac{1}{R \cdot Cf} \cdot \frac{dV_d(t)}{dt} - 2 \cdot w \cdot \frac{dV_q(t)}{dt} - w^2 \cdot V_d(t) - \frac{w}{R \cdot Cf} \cdot V_q(t) + \frac{1}{Lf \cdot Cf} \cdot V_d(t) = \frac{E}{Lf \cdot Cf} \cdot \frac{\sqrt{L_{pri} \cdot L_{\sec}}}{L_{pri}} \cdot D_d(t) \\ \frac{d^2V_q(t)}{dt^2} + \frac{1}{R \cdot Cf} \cdot \frac{dV_q(t)}{dt} + 2 \cdot w \cdot \frac{dV_d(t)}{dt} - w^2 \cdot V_q(t) + \frac{w}{R \cdot Cf} \cdot V_d(t) + \frac{1}{Lf \cdot Cf} \cdot V_q(t) = \frac{E}{Lf \cdot Cf} \cdot \frac{\sqrt{L_{pri} \cdot L_{\sec}}}{L_{pri}} \cdot D_q(t) \end{cases} \quad (10)$$

In the frequency domain, we have:

$$\begin{cases} \left(S^2+\frac{S}{R\cdot Cf}-w^2+\frac{1}{Lf\cdot Cf}\right)\cdot V_d(S)+ \\ +\left(-2\cdot w\cdot S-\frac{w}{R\cdot Cf}\right)\cdot V_q(S)= \\ =\frac{E}{Lf\cdot Cf}\cdot\frac{\sqrt{L_{pri}\cdot L_{\sec}}}{L_{pri}}\cdot D_d(S) \\ \left(S^2+\frac{S}{R\cdot Cf}-w^2+\frac{1}{Lf\cdot Cf}\right)\cdot V_q(S)+ \\ +\left(2\cdot w\cdot S+\frac{w}{R\cdot Cf}\right)\cdot V_d(S)= \\ =\frac{E}{Lf\cdot Cf}\cdot\frac{\sqrt{L_{pri}\cdot L_{\sec}}}{L_{pri}}\cdot D_q(S) \end{cases} \tag{11}$$

IV. DECOUPLING

The dynamic model of the inverter, transformer, filter and load in dq0 coordinates was presented and, since the resulting dq0 inputs of the plant are coupled, a decoupling technique is proposed. Defining the following variables:

$$\begin{cases} a=S^2+\frac{S}{R\cdot Cf}-w^2+\frac{1}{Lf\cdot Cf} \\ b=2\cdot w\cdot S+\frac{w}{R\cdot Cf} \\ c=\frac{E}{Lf\cdot Cf}\cdot\frac{\sqrt{L_{pri}\cdot L_{\sec}}}{L_{pri}} \end{cases} \tag{12}$$

Defining the following equations, we have:

$$\begin{cases} D_d(S)=D_d^*(S)-\frac{b}{c}\cdot V_q(S) \\ D_q(S)=D_q^*(S)+\frac{b}{c}\cdot V_d(S) \end{cases} \tag{13}$$

Therefore:

$$\frac{V_d(S)}{D_d^*(S)}=\frac{E}{Lf\cdot Cf}\cdot\frac{\sqrt{L_{pri}\cdot L_{\sec}}}{L_{pri}}\cdot\frac{1}{S^2+\frac{S}{R\cdot Cf}-w^2+\frac{1}{Lf\cdot Cf}} \tag{14}$$

$$\frac{V_q(S)}{D_q^*(S)}=\frac{E}{Lf\cdot Cf}\cdot\frac{\sqrt{L_{pri}\cdot L_{\sec}}}{L_{pri}}\cdot\frac{1}{S^2+\frac{S}{R\cdot Cf}-w^2+\frac{1}{Lf\cdot Cf}} \tag{15}$$

V. SIMULATION RESULTS

In this section, the PSPICE simulation results of the three-phase inverter with a delta-wye transformer using the Park Transformation are shown.

The results of simulation with the sinusoidal three-phase inverter are presented operating in open loop. At 30 ms instant, a step in the duty cycle of direct axle is applied, with the duty cycle of quadrature axle kept unchanged. Thus the direct and quadrature axles voltages are observed, to prove that they are not coupled.

In the analysis of the open loop voltage a step in the duty cycle of direct axle of 25 % of the nominal value was applied, observing the behavior of the voltages of direct axle and quadrature.

In Fig. 3 they show the voltages feeding the three-phase load. In Fig. 4 the duty cycle of reference are presented, where the step in the 30ms instant is applied.

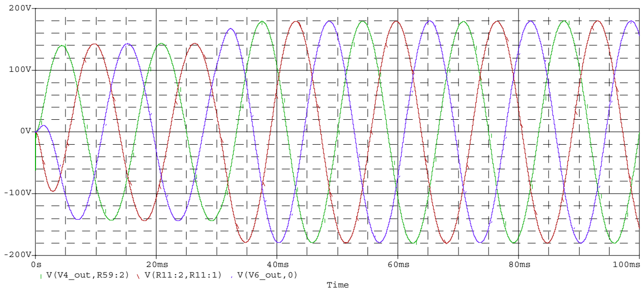

Fig. 3– Three-phase voltages on the Load, working in open loop configuration.

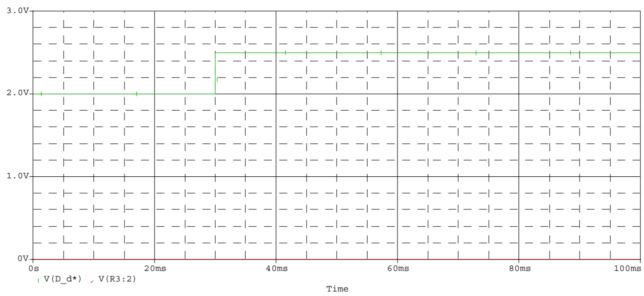

Fig. 4 – Duty cycle of reference.

In Fig. 5 the direct axle and quadrature axle of reference are presented.

In order to test the goodness of the structure on load variations, a change of the load situation was applied from full for practically emptiness no-load (the load was only an electrical resistance of 10k) in one instant of time, making the structure operate practically with a two-phase load. In another instant, however, another load step is applied with the same characteristics of the first one, however in another phase, making the inverter operating with a single-phase load. The objective is to observe if the structure keeps the voltage on the load the desired rms value and a low harmonic content.

TABLE I – Parameters of the Converter

	Specifications
Continuous tension of Entrance	600 *volts*
Efficient Tension of Exit	127 *volts*
Three-phase Power of Exit	10 *kVA*
Frequency of Commutation	7 *kHz*
Capacitor of Filter Passive LC	200 μF
Inductive of Filter Passive LC	1 *mH*
Value of Peak of the Triangular	5 *volts*
Profit of the Sensor of	0,023

Tension. Relation of Transformation.	1

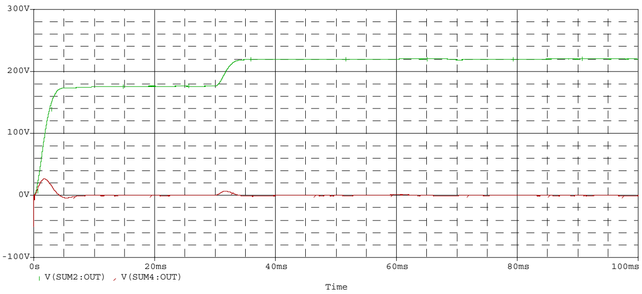

Fig. 5 – Direct and quadrature axle voltages.

A peak voltage of 180 volts. A phase load loss, represented by a variation in its rated value to practically without a load, was emulated at the time that corresponds to 14ms. At 30ms another load was removed and at 60ms the system operates without a load. Figure 6 shows the voltages across the load. Figure 7 shows the waveforms of reference voltages d and q.

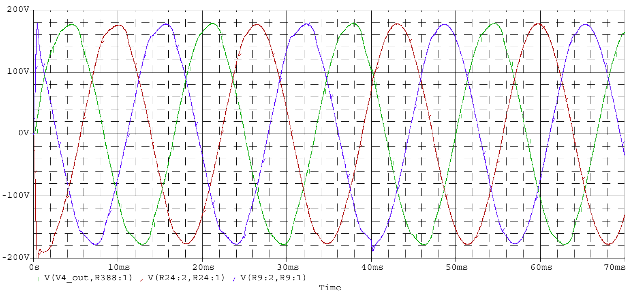

Fig. 6 – Voltages across the three-phase load.

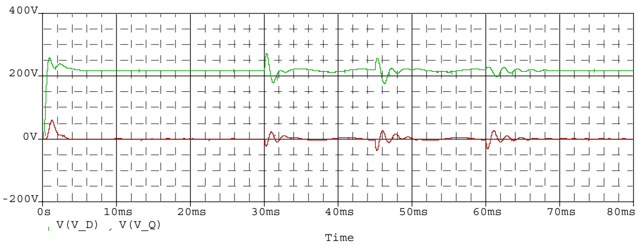

Fig. 7 – Reference voltages d and q.

In Figure 8 the currents in the secondary of the transforming are represented. As it can be observed, in one output the current goes to zero in the instant of corresponding to 14ms, and in another output in the 30ms instant.

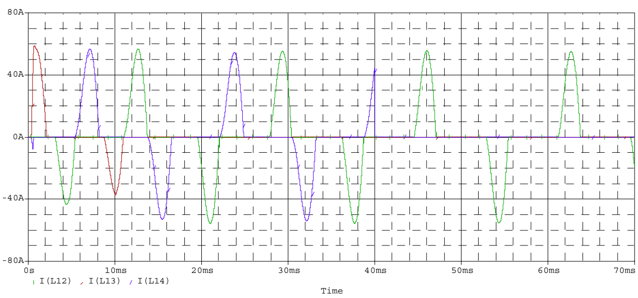

Fig. 8 – Currents in the secondary one of the transformer.

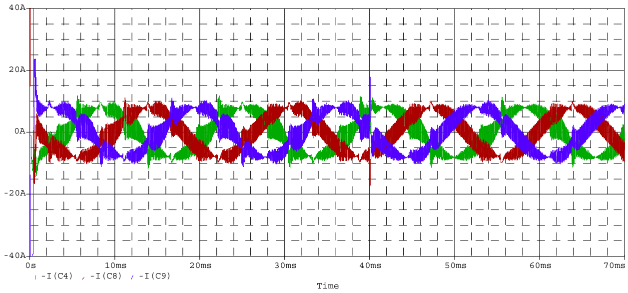

Fig. 9 – Currents in the secondary of the transformer.

In fig. 9 the forms of wave of currents in the capacitors of the LC filter.

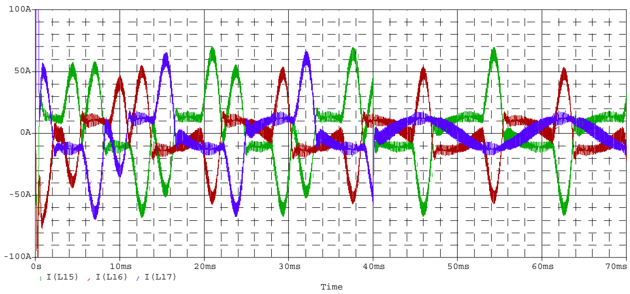

Fig. 10 – Currents in the primary of the transformer.

In Fig. 10 the forms of wave of currents in the primary winding of the transformer reveal the delta connection.

The results of the total harmonic distortion of the phase voltages, for non-linear load are showed in Table II.

TABLE II – Harmonic distortion rate.

	Total Harmonic Distortion (%)
Voltage of phase 1	1,892
Voltage of phase 2	1,789
Voltage of phase 3	1,374

VI. CONCLUSION

The final objective, which was to prove the functionality of the structure, was reached, showing that it is feasible. The converter presented good performance. Simulation results were presented, as well as a theoretical development for the Park Transformation and the filter. The decoupling concepts were also presented.

ACKNOWLEDGEMENT

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