A Control Strategy for Four-Wire Shunt Active Filters Using Instantaneous Active and Reactive Current Method

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***Abstract* – This paper presents a three-phase four-wire active filter control strategy using the instantaneous active and reactive currents. Analyses of the currents including their harmonic components are presented, as well as simulation results that compare the mentioned theories.**

I. INTRODUCTION

Studies dealing with reactive power compensation date from 1976 [1], but the p-q theory was proposed only in 1983 [2]. This theory is valid for any current and voltage waveforms, and is widely used in the control of active filters. The p-q theory is extremely attractive due to the efficiency and relative simplicity, fundamental in times when analog electronics reigned.

Shunt active filters are supposed to compensate the load current implying more balanced and sinusoidal input current. The basis of p-q theory is the control of instantaneous powers, what allows the indirect control of the converter currents. However, the use of dq0 transformation causes the currents to be controlled directly.

This paper presents a control technique that using the dq0 transformation (7) acts in instantaneous active (I_d), reactive (I_q) and zero (I_0) sequence currents.

II. P-Q THEORY

The p-q theory is based on the 0 transformation, also known as the Clark transformation (1).

$$C_{\alpha\beta 0} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & -\sqrt{3}/2 & \sqrt{3}/2 \end{bmatrix} \quad (1)$$

One advantage of this theory is the independence of zero sequence components [3].

Another important definition is the three-phase instantaneous active power (2), that describes the total instantaneous energy flow per time unit between two subsystems, and the three-phase instantaneous reactive power (3), representing the power quantities that do not contribute to the three-phase instantaneous active power.

$$p_{3\phi}(t) = v_1(t) \cdot i_1(t) + v_2(t) \cdot i_2(t) + v_3(t) \cdot i_3(t) \quad (2)$$

$$q_{3\phi}(t) = \frac{1}{\sqrt{3}} \cdot \begin{bmatrix} (v_1(t) - v_2(t)) \cdot i_3(t) + (v_2(t) - v_3(t)) \cdot i_1(t) \\ + (v_3(t) - v_1(t)) \cdot i_2(t) \end{bmatrix} \quad (3)$$

The instantaneous powers defined in the dq0 system are the real power $p(t)$, the imaginary power $q(t)$, and the zero sequence power $p_0(t)$

$$\begin{bmatrix} p_0(t) \\ p(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} v_0(t) & 0 & 0 \\ 0 & v_d(t) & v_q(t) \\ 0 & v_q(t) & -v_d(t) \end{bmatrix} \cdot \begin{bmatrix} i_0(t) \\ i_d(t) \\ i_q(t) \end{bmatrix} \quad (4)$$

Such powers can be divided in continuous and alternated components according to (5).

$$\begin{cases} p_0(t) = \overline{p_0} + \widetilde{p_0} \\ p(t) = \overline{p} + \widetilde{p} \\ q(t) = \overline{q} + \widetilde{q} \end{cases} \quad (5)$$

For balanced systems only the continuous parts $\overline{p}$ and $\overline{q}$ exist, while $p_0(t)$ is null.

The presence of zero sequence components in the voltage and current causes the appearance of power $p_0(t)$, while negative sequence components provide the appearance of alternate powers $\widetilde{p}$ and $\widetilde{q}$.

Fig. 1 shows the physical meaning of instantaneous powers $p(t)$, $q(t)$ and $p_0(t)$.

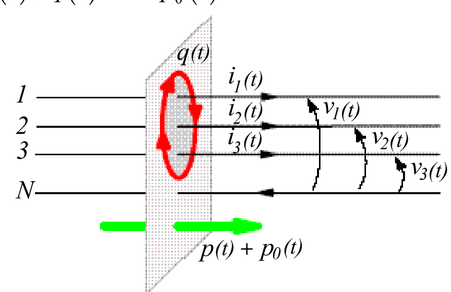

Fig. 1 – Physical meaning of instantaneous powers $p(t)$, $p_0(t)$ and $q(t)$.

Compensating the desired powers, usually $\widetilde{p}$, $q(t)$ and $p_0(t)$, the reference currents are obtained to be used in the controllers.

If the input voltages are distorted, the converter provides distorted currents to the system, since p-q theory is supposed to compensate the power quantities, and not currents. Phase Locked Loop (PLL) is applied to solve this problem, as the fundamental positive sequence component of the input voltage is obtained.

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III. INTANTANEOUS ACTIVE AND REACTIVE CURRENTS

Analyzing a three-phase four-wire system in the dq0 coordinate system i.e. applying the dq0 transformation, the matrix resulting from the product between the Park (6) and the Clarke (1) transformations can be obtained as (7), [4].

$$P_{\alpha\beta 0\rightarrow dq0}=\begin{bmatrix}1 & 0 & 0\\ 0 & \sin(\omega\cdot t) & \cos(\omega\cdot t)\\ 0 & \cos(\omega\cdot t) & \text{-}\sin(\omega\cdot t)\end{bmatrix} \quad (6)$$

$$T_{dq0}=\sqrt{\frac{2}{3}}\cdot\begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2}\\ \sin(\omega\cdot t) & \sin\left(\omega\cdot t-120^{\circ}\right) & \sin\left(\omega\cdot t-120^{\circ}\right)\\ \cos(\omega\cdot t) & \cos\left(\omega\cdot t-120^{\circ}\right) & \cos\left(\omega\cdot t-120^{\circ}\right)\end{bmatrix} \quad (7)$$

The advantages of such transformation are the independence of the zero sequence components; the fundamental positive sequence component is treated as continuous signal; and the negative sequence components are alternate signals with frequency 2ω, where ω is the angular frequency of the power system.

Fig. 2 shows the currents obtained for an unbalanced four-wire load.

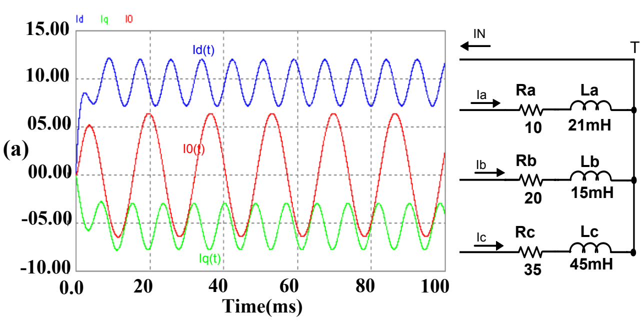

Fig. 2 – Unbalanced four-wire inductive load, $I_d(t)=\overline{I_d}+\widetilde{I_d}$, $I_q(t)=\overline{I_q}+\widetilde{I_q}$ and $I_0(t)=\widetilde{I_0}$.

Currents $I_d(t)$, $I_q(t)$ and $I_0(t)$ present the same characteristics as powers $p(t)$, $q(t)$ and $p_0(t)$. In fact it is more than mere similarity. Analyzing (4) for balanced and sinusoidal voltages i.e. $V_q(t)$ and $V_0(t)$ are null and $V_d(t)=\overline{V_d}$, it can be seen that currents $I_d(t)$ and $I_q(t)$ are responsible for powers $p(t)$ and $q(t)$, respectively.

If the input voltages are unbalanced and/or distorted, the powers will not be compensated as desired, but the currents will. However the main concern is to control the currents.

IV. RELATIONSHIP BETWEEN THE SYMMETRICAL COMPONENTS THEORY AND THE RESULTS OBSERVED IN THE DQ0 BASIS, INCLUDING THE HARMONIC COMPONENTS

A. Analysis of Fundamental Components

The dq0 transformation takes a generic three-phase system to follow a given rotating field with angular speed ω. The fundamental positive sequence components are in phase with such rotating field, and consequently they are treated as continuous signals. The fundamental negative sequence component also creates a rotating field with angular speed ω, but in the opposite sense, so that the relative angular speed is 2ω. The fundamental zero sequence component presents angular speed ω. Fundamental positive, negative and zero sequence components are noticed in Fig. 2.

B. Analysis of Harmonic Components

Additional analysis can be performed for the harmonics symmetrical components according to Table I. The symmetrical components for higher orders can be obtained analogously.

TABLE I
HARMONICS SEQUENCE

Order	1	2	3	4	5	6	7
Seq.	+	-	0	+	-	0	+

To understand the phenomenon observed when harmonics are analyzed with the dq0 transformation, it is necessary to apply the same analogy used for the symmetrical components in the fundamental frequency. Three different cases will be presented as follows, as the remaining ones are redundant.

- **First case: harmonics with negative symmetrical components**. As an example, the second harmonic is chosen. The angular speed of the rotating field generated by the currents is 2ω, but in the opposite sense to the rotating field generated by the dq0 transformation, as the relative angular speed is 3ω. The relationship between the amplitudes of currents $I_d(t)$ and $I_q(t)$ and the conventional currents is $\sqrt{3/2}$. Fig. 3 presents the simulation results for the first case;

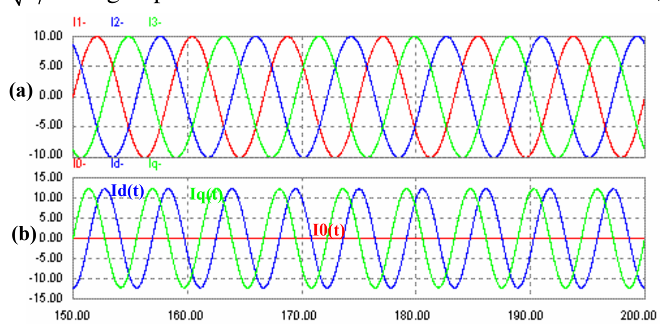

Fig. 3 – First case: (a) Second harmonic currents and (b) Resulting currents with dq0 transformation.

- **Second case: harmonics with zero symmetrical components.** As an example, the third harmonic is chosen. The frequency of the currents remains constant (at 180Hz in this case), but the amplitude is $\sqrt{3}$ times greater. Fig. 4 presents the simulation results for the second case;

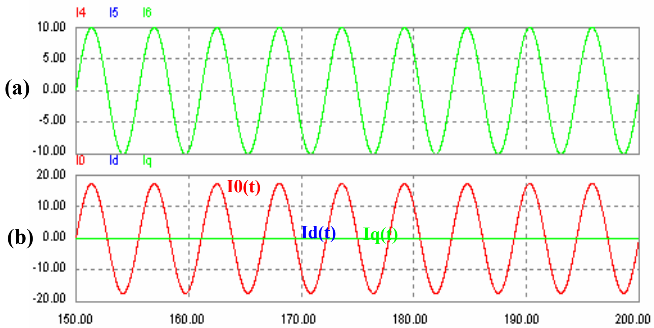

Fig. 4 – Second case: (a) Third harmonic currents and (b) Resulting currents with dq0 transformation.

- Third case: harmonics with positive symmetrical components. As an example, the fourth harmonic is chosen. The angular speed of the rotating field generated by the currents is 4ω, but in the same sense of the rotating field generated by the dq0 transformation, as the relative angular speed is 3ω. The relationship between the amplitudes of currents $I_d(t)$ and $I_q(t)$ and the conventional currents is $\sqrt{3/2}$. Fig. 5 presents the simulation results for the third case.

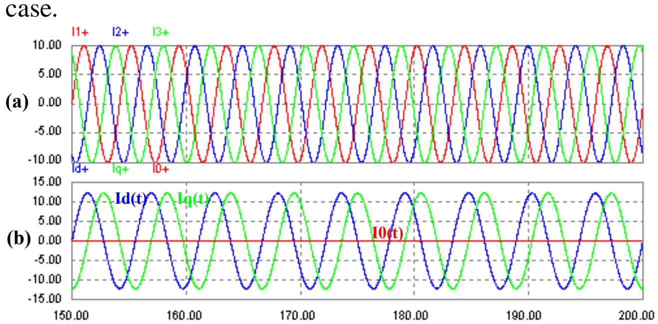

Fig. 5 – Third case: (a) Fourth harmonic currents and (b) Resulting currents with dq0 transformation.

V. THREE-PHASE FOUR-WIRE PWM CONVERTER WITH SPLIT-CAPACITOR AND CONTROL STRUCTURE

A. Three-Phase Four-Wire PWM Converter with Split-Capacitor

In order to obtain simulation results and illustrate the subsequent statements, the converter presented in Fig. 6 was designed using the dq0 transformation. The transfer functions of the converter were obtained, between the input current and the duty cycle (8), and between the output voltage and the input current with the dq0 transformation (9), [5].

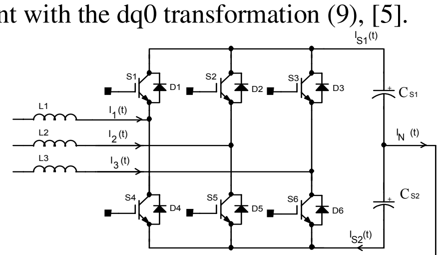

Fig. 6 – Three-phase four-wire PWM converter with split-capacitor.

$$\begin{cases} \dfrac{i_0(s)}{\tilde{d}_0(s)} = -\dfrac{V_S}{L \cdot s + R_S} \\ \dfrac{i_d(s)}{\tilde{d}_d'(s)} = -\dfrac{V_S}{L \cdot s + R_S} \\ \dfrac{i_q(s)}{\tilde{d}_q'(s)} = -\dfrac{V_S}{L \cdot s + R_S} \end{cases} \tag{8}$$

$$\begin{cases} \dfrac{V_S(s)}{i_d(s)} = \dfrac{\sqrt{6} \cdot V_{pk}}{s \cdot V_S \cdot C_S} \cdot \left[1 - \left[\dfrac{2 \cdot S_p \cdot (1-\eta) \cdot (2 \cdot R_S + s \cdot L)}{3 \cdot V_{Pk}^2} \right] \right] \\ \dfrac{V_S(s)}{i_q(s)} = -2 \cdot \sqrt{\dfrac{2}{3}} \cdot \dfrac{S_p}{V_{Pk} \cdot V_S} \cdot \left[\dfrac{L \cdot s + 2 \cdot R_S}{s \cdot C_S} \right] \\ \dfrac{V_S(s)}{i_0(s)} = \dfrac{\sqrt{3}}{s \cdot C_S} \end{cases} \tag{9}$$

where V_S is the output voltage, R_S is the series equivalent resistance for each one of the three branches, L is the boost inductance, V_{pk} is the peak input voltage, C_S is the output capacitance, η is the converter efficiency, and S_p is the complex power processed by the converter.

B. Control Structure

Fig. 7 shows the main block diagram, where the converter is placed in the secondary side of a distribution transformer, in parallel with a four-wire unbalanced load. The converter is supposed to compensate the unbalance as well as the harmonic components of the load current.

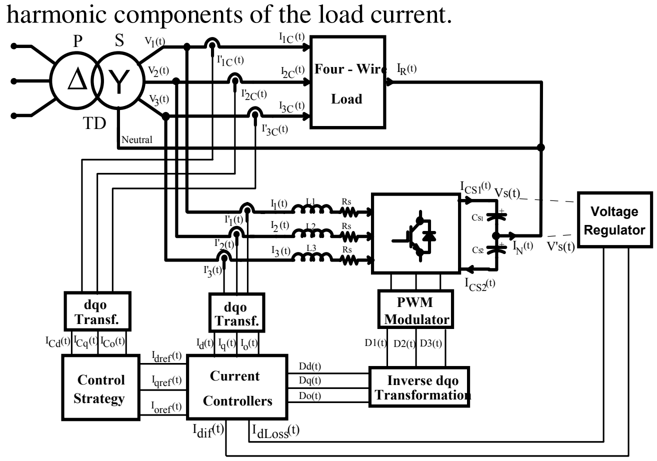

Fig. 7 – Main block diagram.

Two control strategies will be presented in this work. The first one considers the compensation of instantaneous powers and the second one deal with the compensation of currents. The transformation is synchronized with the zero crossing voltage detector. If the voltages are balanced with only positive sequence components, the strategies will provide identical results.

According to Fig. 8 (a), the control system is based on the dq0 transformation applied to the converter currents, which are taken to the compensators in Fig. 8 (b).

The total output voltage and the voltage across capacitor C_{S2} are measured and applied to the voltage regulator block, composed by two distinct controllers. The first one controls the total output voltage, compensating the converter losses, represented by current $I_{dLoss}(t)$. The second one corrects the voltage across the output capacitors, as the output signal is called difference current $I_{Dif}(t)$.

Both currents are applied to the current controllers, since $I_{dLoss}(t)$ and $I_{Dif}(t)$ are added to $I_{dref}(t)$ and $I_0(t)$, respectively. The voltage regulator is shown in Fig. 9 (a), where gains and low-pass filters are used to eliminate the alternate components of the voltages.

The output signals of the current controllers are the duty cycles in dq0 coordinates. The inverse dq0 transformation is used in Fig. 9 (b), as duty cycles $d_1(t)$, $d_2(t)$ and $d_3(t)$ are applied to the modulator block.

The voltage and current compensators are just PI controllers.

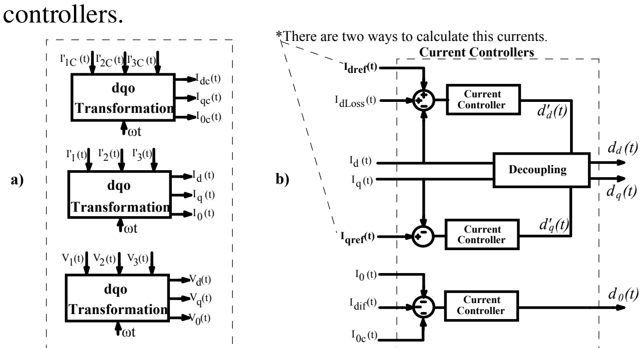

Fig. 8 – (a) Dq0 transformation of load currents, converter currents and input voltages; (b) Current controllers.

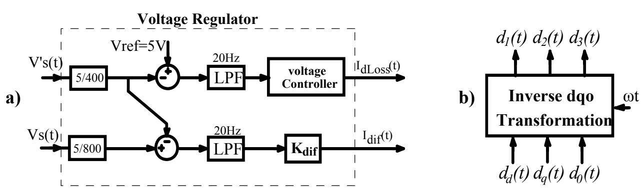

Fig. 9 – (a) Voltage regulator; (b) Inverse dq0 transformation.

The adopted control strategy defines the reference currents. Firstly, the load currents must be measured and converted to the dq0 system. Fig. 10 (a) shows the reference currents calculated from the load currents. Currents $\tilde{I}_{dc}(t)$ and $\tilde{I}_{qc}(t)$ are alternate parts of sequence components d and q, respectively.

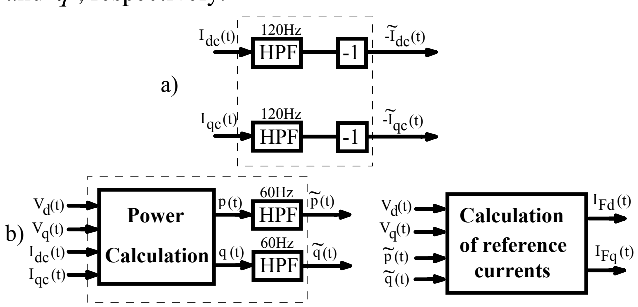

Fig. 10 – Two different control strategies: (a) Considering the currents as reference (b) Considering the instantaneous powers as reference.

Fig. 10 (b) presents the calculation of the reference currents considering the instantaneous powers. The input voltage is also measured and taken to the dq0 system. From the input voltages and load currents, instantaneous powers $p(t)$ and $q(t)$ can be calculated.

The alternate components of the currents and powers are obtained by the application of high-pass filters. The powers are calculated from (10), and the reference currents are obtained from (11). Reference current $I_{0ref}(t)$ is the same one for both methods i.e. it is the zero sequence component of load current $I_{0c}(t)$ with negative sign.

$$\begin{bmatrix} p(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} v_d(t) & v_q(t) \\ v_q(t) & -v_d(t) \end{bmatrix} \cdot \begin{bmatrix} i_{dc}(t) \\ i_{qc}(t) \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} I_{Fd}(t) \\ I_{Fq}(t) \end{bmatrix} = \frac{-1}{v_d(t)^2 + v_q(t)^2} \begin{bmatrix} v_d(t) & v_q(t) \\ v_q(t) & -v_d(t) \end{bmatrix} \cdot \begin{bmatrix} \tilde{p}(t) \\ \tilde{q}(t) \end{bmatrix} \tag{11}$$

VI. SIMULATION RESULTS

In order to compare the aforementioned theories and to validate the theoretical assumptions about the instantaneous current control, simulation results are presented. The parameters set used in the design are shown in Table I.

TABLE I
DESIGN PARAMETERS

Rms line voltage (V_{ac})	220Vrms
Line frequency (f_L)	60 Hz
Output voltage (V_S)	800V
Switching frequency (f_S)	20kHz
Boost inductor (L)	700μH
Output capacitor (C_S)	14.1mF

The unbalanced voltages given by (12) are used in the converter simulation.

$$\begin{cases} v_1(t) = 310 \cdot \sin(\omega t + 8^\circ) \\ v_2(t) = 325 \cdot \sin(\omega t - 117^\circ) \\ v_3(t) = 300 \cdot \sin(\omega t + 123^\circ) \end{cases} \tag{12}$$

A. Applying the P-Q Theory

Fig. 11 shows input voltages, input currents, load currents and converter currents. The input current is visibly distorted due to the control strategy.

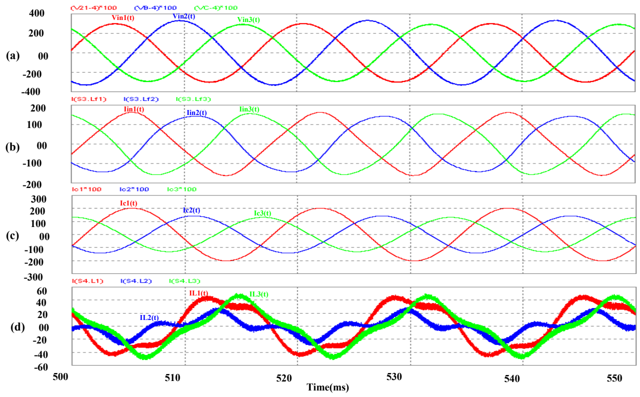

Fig. 11 – (a) Input voltages; (b) Input currents; (c) Load currents and (d) Converter currents.

Fig. 12 represents the currents in dq0 coordinates regarding input, load and converter. It can be seen that the alternate components of currents d and q are not compensated satisfactorily. The zero sequence current is practically null in both cases.

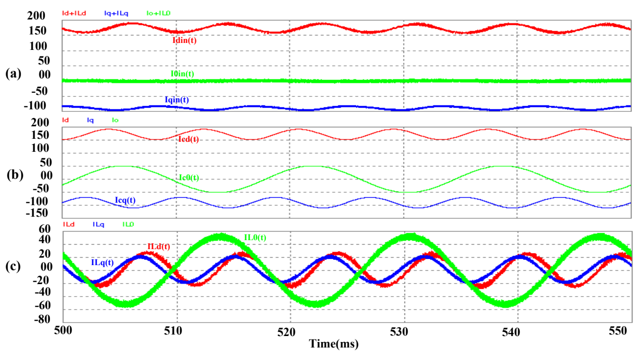

Fig. 12 – Currents in the dq0 system: (a) Input current; (b) Load current and (c) Converter current.

Fig. 13 presents the three-phase instantaneous active and reactive powers for input, load and converter. The converter practically compensated the alternate components in the three-phase instantaneous powers.

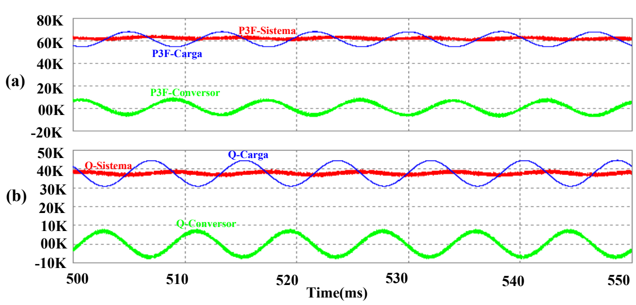

Fig. 13 – Three-phase instantaneous powers: (a) Active power; (b) Reactive power.

Table II shows the total harmonic distortion of the input current.

TABLE II
TOTAL HARMONIC DISTORTION OF THE INPUT CURRENT

	Iin1	Iin2	Iin3
THD(%)	5.615	5.668	5.72

B. Simulation Applying the Instantaneous Active and Reactive Current

Fig. 14 presents the input voltages, input currents, load currents and converter currents.

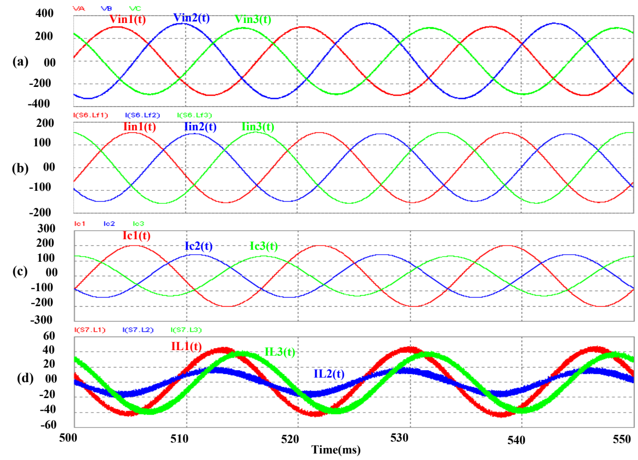

Fig. 14 – (a) Input voltages; (b) Input currents; (c) Load currents and (d) Converter currents.

In Fig. 15 one can see the currents in dq0 system for input, load and converter. Components d and q of the input current are practically constant, and the zero sequence current is almost null. The converter processes the alternate part of the load current with the same amplitude.

Table III shows the total harmonic distortion of the input current.

TABLE III
TOTAL HARMONIC DISTORTION OF THE INPUT CURRENT

	Iin1	Iin2	Iin3
THD(%)	0.377	0.382	0.441

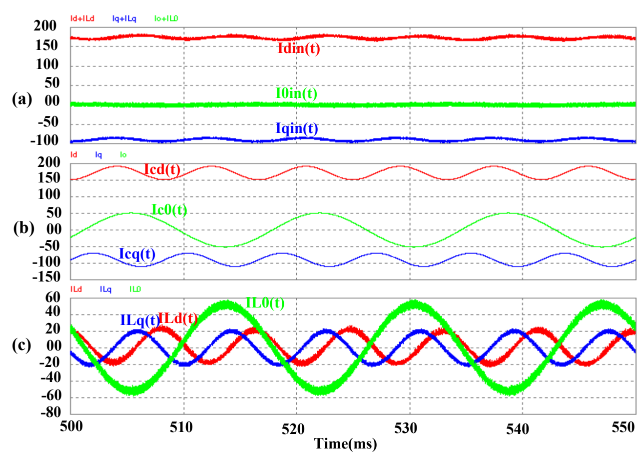

Fig. 15 – Currents in dq0 system: (a) Input current; (b) Load current and (c) Converter current.

Fig. 16 presents the instantaneous active and reactive three-phase powers for the input, load, and converter. The converter is not able to compensate the alternate components in the three-phase instantaneous powers satisfactorily.

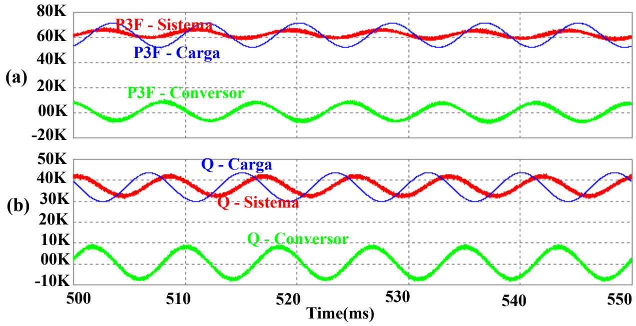

Fig. 16 – Three-phase instantaneous powers: (a) Active power; (b) Reactive power.

Fig. 17 presents simulation results when the converter is used in the compensation of nonlinear loads.

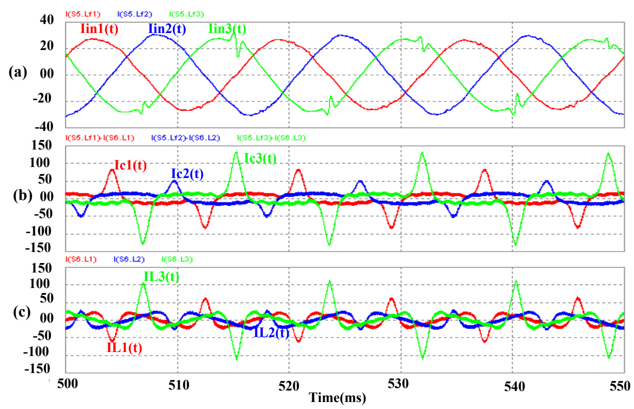

Fig. 17 – Nonlinear loads: (a) Input current; (b) Load current and (c) Converter current.

It can be seen that this technique allows the separation of the currents according to the components that will be compensated e.g. reactive components, harmonics, negative or zero sequence components.

VII. CONCLUSION

This paper has presented two theories for the calculation of the reference currents in a shunt active filter. The first one is the very well known p-q theory, as the input currents can be significantly distorted if the PLL is not used. The second one, the instantaneous active and reactive current, provides balanced currents with low harmonic distortion using only a zero crossing voltage detector.

If the input voltages are balanced identical results are obtained.

VIII. ACKNOWLEDGMENT

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IX. REFERENCES

[1] L. Gyugyi, E. Strycula, “Active AC Power Filters”, IEEE IAS Annual Meeting, pp. 529-535, 1976.

[2] H. Akagi, Y. Kanazawa, A. Nabae, “Instantaneous Reactive Power Compensators Comprising Switching Devices without Energy Storage Components”, IEEE Transactions on Industry Application, vol. IA-20, pp. 625-630, 1984.

[3] M. Aredes, “Active Power Line Conditioners”. Dr.-Ing. Thesis, Technischen Universität Berlin, Berlin, Germany, March 1996.

[4] P. Verdelho, G. Marques, “An active power filter and unbalanced current compensator”, *IEEE Trans. Ind. Electron.* Vol. 44, p. 321-328, June 1997.

[5] D. Borgonovo, I. Barbi, Y. R. Novaes. “A Three-Phase Three-Switch Two-Level PWM Rectifier”, Record of the 34th IEEE Power Electronics Specialists Conference, vol. 3, June 2003, pp. 1075-1079.