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Denizar C. Martins Non-member  
Luis C. Tomaselli Non-member  
Telles B. Lazzarin Non-member  
Ivo Barbi Non-member

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# Drive for a Symmetrical Two-Phase Induction Machine Using Vector Modulation

Denizar C. Martins\* Non-member  
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In this paper a drive for a symmetrical two phase induction machine using vector modulation is studied. A comparison is made among different types of space vector modulation (discontinuous and continuous) and a new one is proposed. The concepts of vector modulation are reviewed. Experimental results to validate the theoretical statements are shown.

**Keywords:** space vector modulation, two phase induction symmetrical motor, AC drive

## 1. Introduction

Driving two-phase machines is of great interest to industries due to their potential for residential applications (compressors, fans, etc.). One of the known methods of varying the speed of a single-phase induction motor with a permanent capacitor (PSC motor) is to remove the capacitor from the auxiliary winding and consider it to be a two-phase machine<sup>(1)(2)(6)</sup>. In this way, the motor would be fed by a two-phase inverter. The problem is that the windings of the motor are not identical, which makes feeding the two phases with different voltages a necessity. This asymmetry is due to the presence of the capacitor, which forms a resonant circuit with the motor's inductance and, consequently, raises the voltage across the auxiliary winding. Thus, instead of designing a two-phase inverter to feed the PSC motor without a capacitor, it is better to redesign the motor so that it becomes a symmetrical two-phase machine. The advantage is that the motor is now fed by balanced voltages and, therefore, makes better use of the voltage available at the DC bus.

This work focuses on the use of two-phase inverters employing space vector modulation to drive symmetrical two-phase machines. There are three possibilities for the voltage inverter<sup>(4)</sup>. The first is a two phase H-bridge voltage inverter composed of two legs and a mid-point obtained by means of a capacitive divider (Fig. 1). This converter has the disadvantage of not fully using the DC bus, besides presenting voltage balancing problems, especially at low frequencies. The small number of switches is an advantage; however, the volume of the structure and consequently the cost increase due to the capacitors. The second possibility is the two-phase inverter with three legs, in which the mid-point is obtained through the appropriate control of the common leg (Fig. 2). In this case, common mode voltages or co sinusoidal functions with different phases<sup>(7)</sup> can be used as long as carrier

signal modulations are employed. Therefore, the linear operating region of the inverter is extended. The inverter also allows the use of the space vector modulation since it is now possible to use zero vectors. There is an increase in the number of switches when compared to the previous solution. The last possible choice is the four legged inverter, one leg for each phase (Fig. 3). This solution presents the largest linear operation region. However, it is also the solution that presents the greatest number of switches which implies greater losses and more command circuits. This work opted for the two

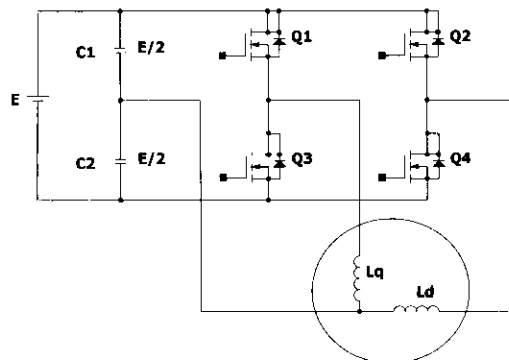


Fig. 1. Two-phase H-bridge voltage inverter

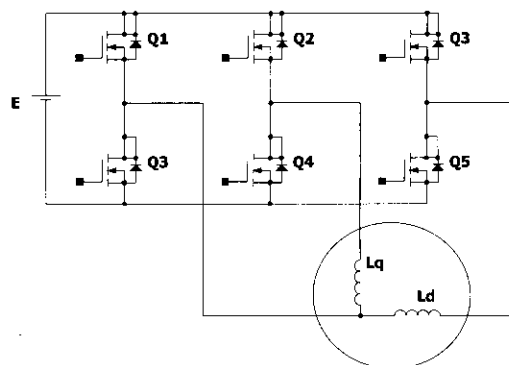


Fig. 2. Two-phase voltage inverter with three legs

\* UFSC-Federal University of Santa Catarina  
INEP-Power Electronics Institute  
P.O. Box 5119-88040-970-Florianopolis, SC-Brazil

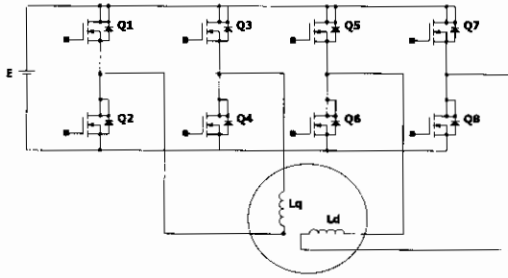


Fig. 3. Two-phase voltage inverter with four legs

phase three-legged voltage inverter.

The main idea of this work is to present a new discontinuous space vector modulation strategy to minimize the losses of the two-phase voltage inverter with three legs.

## 2. Space Vector Modulation Applied to the Two Phase Three-Legged Inverter

Space vector modulation is characterized by not presenting a carrier signal. One of the main advantages of this modulation is the degree of freedom in distributing the time associated with the zero vectors. Using this fact appropriately it can minimize the switching losses. The main disadvantages of this type of modulation are: time precision, there is no minimum time for pulse duration, it does not prevent notches, and it is not easy to apply the overmodulation technique (due to the transition of the modulation function, which is continuous, to a discontinuous function).

Differently from the three-phase case (without a neutral connection it is also possible to obtain a plane representation since one phase is linearly dependent), the six voltage vectors do not form a regular hexagon<sup>(3)</sup> (Fig. 4). From the figure, it is only possible to reproduce at the output a reference vector ( $U_0$ ), which is contained inside the region outlined by the voltage vectors. The linear region is then defined as the region in which the reference vector can be reproduced at the output without distortions.  $U_0$  is then a vector that spins at a constant angular frequency and whose trajectory forms a circumference (because two sinusoids phase shifted ninety degrees from each other, are desired). The amplitude of the reference vector is the radius of this circumference. The linear region is, therefore, defined by the maximum radius of this circumference, obtained from the trajectory of the reference vector, circumscribed by the hexagon formed by the voltage vectors. From Fig. 4 it is possible to determine the maximum value of the radius for the trajectory of the vector in the linear operating region, which is  $1/\sqrt{2}$  pu (normalized value as a function of the DC bus voltage). This is also the maximum value for the reference vector in the linear operating region (observe that this is a relative value). For larger values, the operating region is non-linear.

It can be said that space vector modulation combines the different voltage vectors in order to obtain the desired reference vector at the output of the converter. This is achieved by equalling the average value of the instantaneous vectors during one switching period to the value of the reference vector, as is done in sinusoidal modulation. There are many possible combinations that satisfy this condition, however only the possibilities with the smallest number of switching cycles

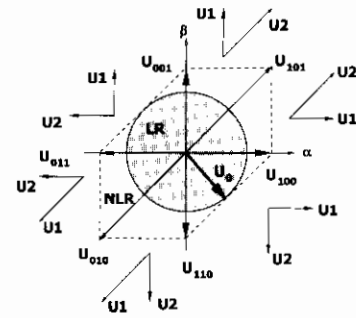


Fig. 4. Hexagon with the six voltage vectors characterizing the VSI operation

are used. It can be observed by connecting the vertices of the hexagon that six right triangles are formed. These triangles define six operating regions or six sectors. The adjacent sides and the hypotenuses form the basic vectors and these are adjacent to any reference vector inside the section. Notice that changing from one vector to another, inside the same section, involves only one switching cycle, in other words, only one of the legs switches. However, when analyzing vectors of different sections this is no longer true. Therefore, one tries to build the reference vector using only the adjacent vectors of the sections. The adjacent vectors of each section are named  $U_1$  and  $U_2$ . These names are preserved when changing sections. In this manner, upon defining  $U_1$  as the vector formed by combination 100, the next vector (in either direction, clockwise or counter clockwise) will be  $U_2$  and, from there, consecutively, the vectors alternate between themselves until they complete the voltage vectors (Fig. 4). In the same way, the time spent on vector  $U_2$  is called  $T_2$  and  $T_1$  is the time associated with  $U_1$ .

Therefore, given a reference vector located within the section defined by  $U_1$  and  $U_2$  (which are both basic adjacent vectors), it is possible to establish the equation that defines the relationship between the times and vectors:

$$T_{PWM} \cdot \vec{U}_0 = T_1 \cdot \vec{U}_1 + T_2 \cdot \vec{U}_2 \dots \dots \dots (1)$$

Expression (1) shows that the product of voltage and time must be equal, that is, the product of the reference vector and the switching period must be equal to the sum of the products of the vectors and their respective times. Thus, the average evolution within the period can be controlled. The sum of times  $T_1$  and  $T_2$  is not always equal to unity, therefore it is necessary to spend the rest of the period on a zero vector, in other words, a vector that does not modify the sum defined by equation (1). Since the sum of the times ( $T_1$  and  $T_2$ ) is smaller than the switching period, to maintain the output voltage constant, zero vectors are used during the rest of the period. So:

$$T_0 = T_{PWM} - T_1 - T_2 \dots \dots \dots (2)$$

By calculating the times, it is possible to generate the switching signals. There are many switching patterns that can be used to implement space vector modulation. One possible sequence of commands is  $U_{000} - U_1 - U_2 - U_{111} - U_2 - U_1 - U_{000}$  and means that the outputs switch sequentially from  $U_{000}$  to  $U_1$ ,  $U_2$ ,  $U_{111}$ ,  $U_2$ ,  $U_1$  and return again to  $U_{000}$  each period, where  $U_1$  and  $U_2$  are basic vectors that form the section in

Table 1. Definitions for Vectors  $U_1$  and  $U_2$

Sector	$U_1$	$U_2$	$V_1$	$V_2$	$\theta_1$	$\theta_2$
1	$U_{100}$	$U_{101}$	1	$\sqrt{2}$	0	45
2	$U_{001}$	$U_{101}$	1	$\sqrt{2}$	90	45
3	$U_{001}$	$U_{011}$	1	1	90	180
4	$U_{010}$	$U_{011}$	$\sqrt{2}$	1	225	180
5	$U_{010}$	$U_{110}$	$\sqrt{2}$	1	225	270
6	$U_{100}$	$U_{110}$	1	1	0	270

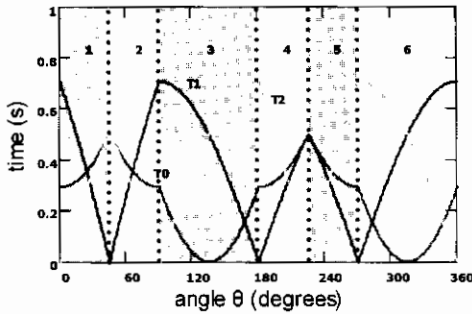


Fig. 5. The times (normalized) associated with each basic vector as a function of angle  $\theta$  of the reference vector

which the reference vector finds itself. It can be said that from time distribution  $T_0$ , during a switching period between the zero vectors, the different space vector modulations appear.

Establishing the notation for the basic vectors:

$$\left. \begin{aligned} \vec{U}_1 &= V_1 e^{j\theta_1} \\ \vec{U}_2 &= V_2 e^{j\theta_2} \end{aligned} \right\} \dots\dots\dots (3)$$

From Fig. 4,  $V_1$ ,  $V_2$ ,  $\theta_1$ , and  $\theta_2$  can be defined for each section. The counter clockwise direction is used to define the angles. Therefore

Supposing that  $U_0 = V e^{j\theta}$ , substituting (3) in (1), separating the real and imaginary parts, writing in matrix notation and isolating  $T_1$  and  $T_2$ :

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = T_{PWM} \begin{bmatrix} V_1 \cos(\theta_1) & V_2 \cos(\theta_2) \\ V_1 \sin(\theta_1) & V_2 \sin(\theta_2) \end{bmatrix}^{-1} \begin{bmatrix} V \cos(\theta) \\ V \sin(\theta) \end{bmatrix} \dots\dots\dots (4)$$

Therefore:

$$\frac{T_1}{T_{PWM}} = \frac{V}{V_1} \cdot \frac{\sin(\theta_2 - \theta)}{\sin(\theta_2 - \theta_1)} \dots\dots\dots (5)$$

$$\frac{T_2}{T_{PWM}} = \frac{V}{V_2} \cdot \frac{\sin(\theta - \theta_1)}{\sin(\theta_2 - \theta_1)} \dots\dots\dots (6)$$

$T_0$ , on the other hand, is defined by (2).  $V_1$ ,  $V_2$ ,  $\theta_1$ , and  $\theta_2$  are defined as functions of the operating section. Expressions (2), (5), and (6) are represented graphically in Fig. 5 for a reference vector of amplitude  $1/\sqrt{2}$  pu and for a complete trajectory of the reference vector. Independent of the space vector modulation used, these times are always constant and only change if the amplitude of the reference voltage is modified.

### 3. Proposed Discontinuous Space Vector Modulation

It is interesting to observe that this modulation presents one

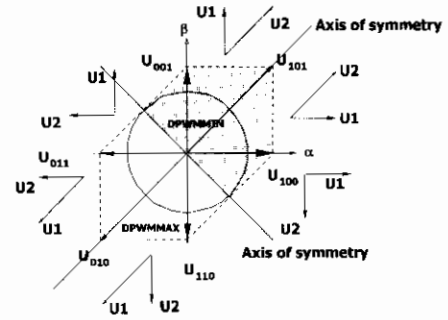


Fig. 6. Switching sequence for each section of the hybrid discontinuous space vector modulation

degree of freedom: time distribution  $T_0$  in both zero vectors. This degree is what originates the different types of modulations. In three-phase inverters, zero time distribution is used between zero vectors during a switching period to specify the type of space vector modulation applied.  $T_{01}$  is defined for zero vector 000 and  $T_{02}$  for zero vector 111. A variable is used to classify the different modulations, given by the following expression <sup>(5)</sup>:

$$\mu = \frac{T_{01}}{T_{01} + T_{02}} \dots\dots\dots (7)$$

When  $\mu$  assumes values between zero and one, the modulation is continuous. When in certain sections  $\mu$  equals the extremes (0 or 1), discontinuous modulation occurs. Discontinuous modulation has the advantage of reducing the number of switching cycles within a period of the reference signal because one leg is always connected to either the positive or the ground terminal of the DC bus.

Thus, when  $T_{01} = T_{02}$ ,  $\mu = 0.5$ , continuous space vector modulation (CSVPWM) occurs. When  $T_{01} = T_0$  and  $T_{02} = 0$ ,  $\mu = 1$ , minimum discontinuous space vector modulation (DPWMMIN) occurs. However, when  $T_{01} = 0$  and  $T_{02} = T_0$ ,  $\mu = 0$ , maximum space vector modulation (DPWMMAX) occurs. Discontinuous modulation is characterized by the fact that one of the legs is not switching. In the three-phase case, each leg remains inactive during 120 degrees. In the two-phase case, the common leg remains inactive during 90 degrees while the other two legs are inactive during 135 degrees. Here, the thermal disequilibrium caused by improper current distribution among the switches (as described further ahead) is added to the effect of a switching frequency different from that of the common leg in relation to the other legs. Furthermore, the common leg conducts more current and, as a reminder, the switching losses are proportional to current and frequency. To avoid these problems, this work proposes a new technique. Axes of symmetry (shifted 45 degrees from the axes of plane DQ) are defined to divide the hexagon into two equal parts, one using DPWMMIN and the other using DPWMMAX, in other words, for 180 degrees  $\mu = 1$  and during the other 180 degrees  $\mu = 0$ . This is called hybrid discontinuous space vector modulation (Fig. 6). Using this strategy, the common leg does not switch during 180 degrees while the other legs are inactive during 90 degrees (this is good because the common leg conducts the most current). In the same way, the disequilibrium between the switches of a leg is eliminated.

The behaviour of the duty cycles of the legs (upper switch)

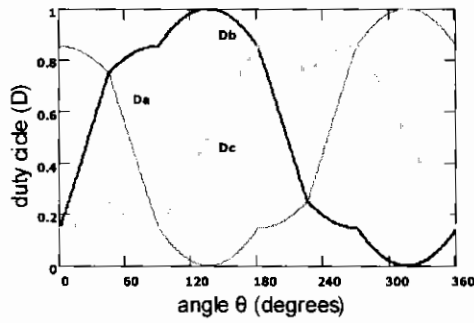


Fig. 7. Duty cycle of the legs using standard space vector modulation (CSVPWM)

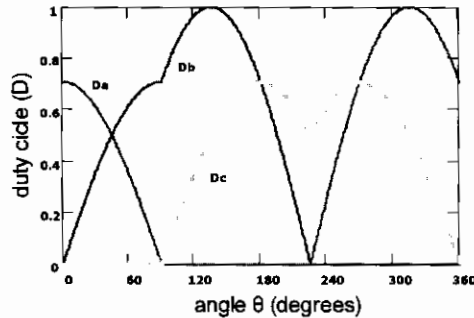


Fig. 8. Duty cycle of the legs using minimum discontinuous space vector modulation (DPWMMIN)

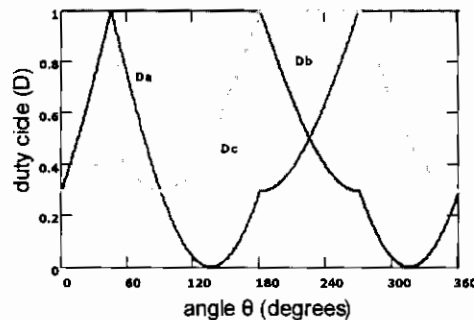


Fig. 9. Duty cycle of the legs using maximum discontinuous space vector modulation (DPWMMAX)

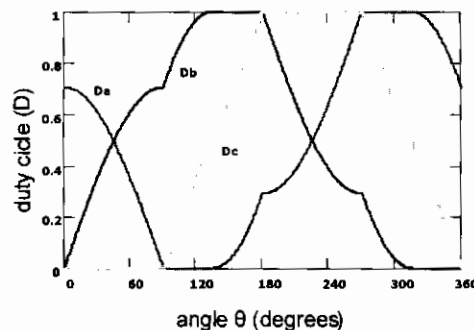


Fig. 10. Duty cycle of the legs using hybrid discontinuous space vector modulation (DPWMHIB)

is presented in Figures 7, 8, 9, and 10 using the following modulations, respectively: CSVPWM, DPWMMIN, DPWMMAX, and DPWMHIB.

It is important to remember that continuous and hybrid space vector modulations present better current distribution among the switches when compared to discontinuous modulations. In minimum and maximum discontinuous

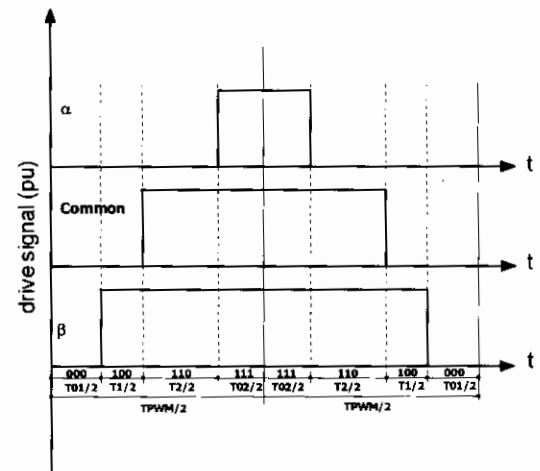


Fig. 11. Symmetrical continuous space vector modulation

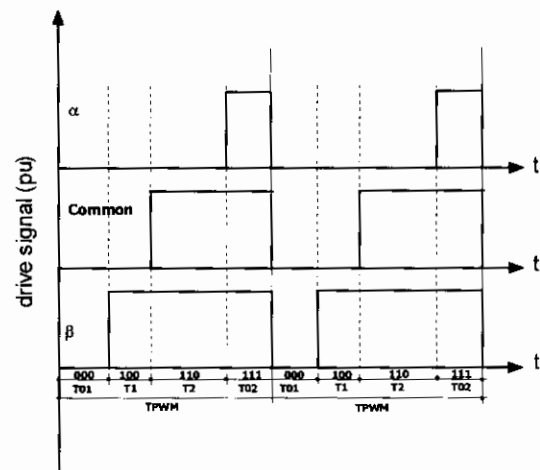


Fig. 12. Asymmetrical continuous space vector modulation

modulations, either the top or bottom switches conduct longer. In terms of current distribution, continuous space vector modulation is better. The advantage of discontinuous space vector modulation resides in the fact that the switching losses are limited to one third of the switching losses of continuous space vector modulation. Hybrid discontinuous space vector modulation, of all, is the most similar to continuous space vector modulation (when referring to the distribution of conduction losses in a leg).

There is still the possibility of using all of these modulations symmetrically (Fig. 11) and asymmetrically (Fig. 12). In symmetrical modulation there is symmetry at half the switching period which does not occur in asymmetrical modulation. A correlation between pulse width modulations, obtained from a triangular and a saw-tooth waveform, can be done.

#### 4. Experimental Results

A prototype (Fig. 13) was implemented to study this driving. A two-phase symmetrical motor with IV poles and operating at 220 V, 60 Hz, and 350 W of rated power was used. The switching frequency used was 5 kHz (a limitation imposed by the microcontroller). Figures 14, 15, and 16 present some experimental results (stator currents and the voltages across the legs in detail).

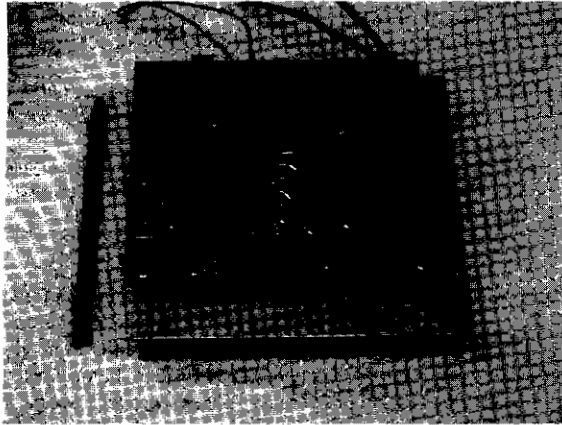


Fig. 13. Photograph of the implemented prototype

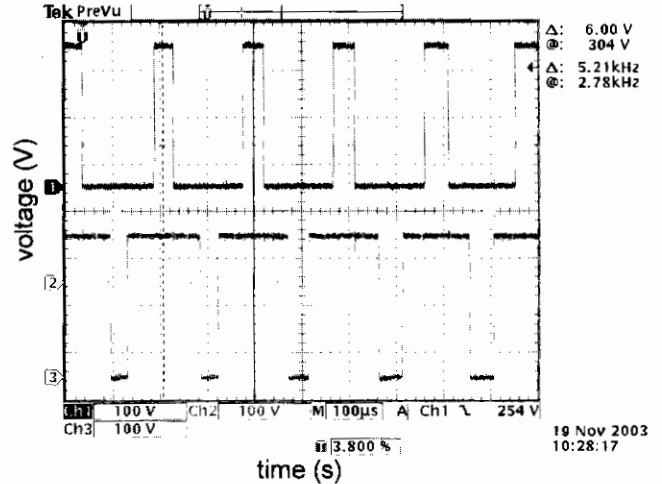


Fig. 16. Voltages across the legs in detail

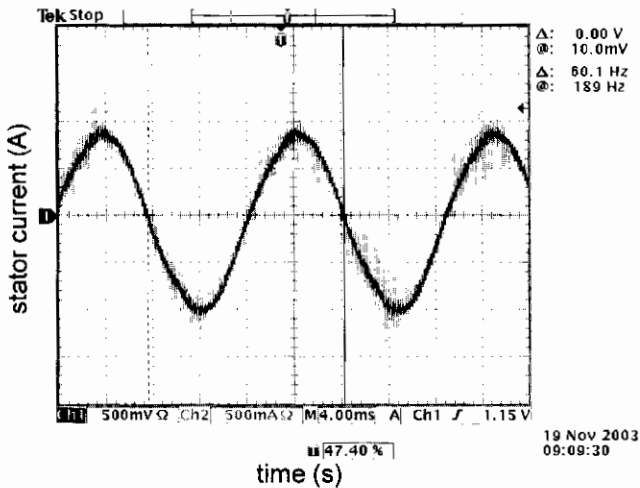


Fig. 14. 60 Hz stator currents

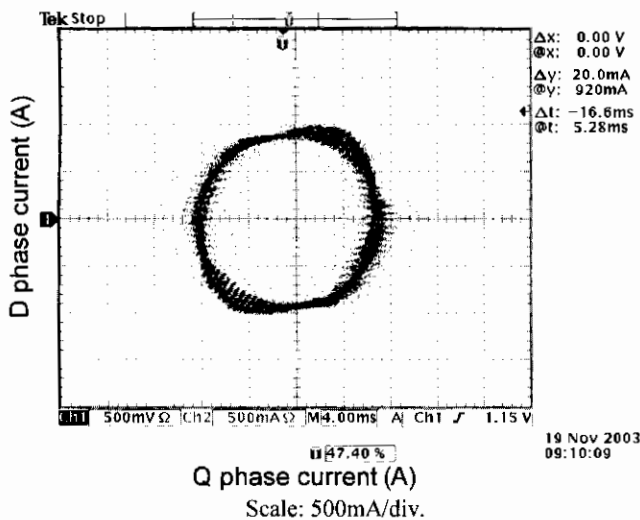


Fig. 15. 60 Hz stator currents in the DQ plane

At low frequencies, the currents are deformed. This is due to the manner in which the modulation was implemented. Since the modulation index at low frequencies decreases, the times associated with the non-zero vectors are small, approaching, in value, to the delay time of the program. Therefore, small times are limited to a specified minimum value. This causes deformations in the current. Another important

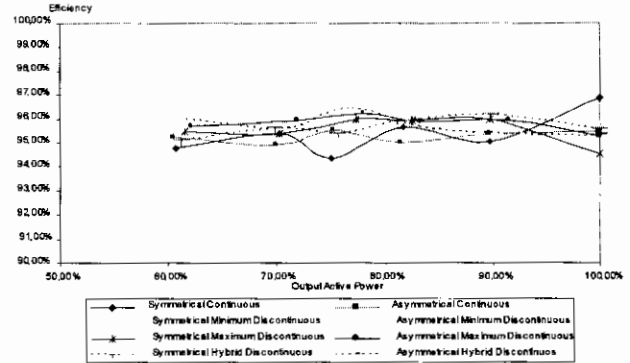


Fig. 17. Experimentally obtained efficiency for the inverter with a two phase load (RL type) versus the variation of the output power (normalized as a function of the rated value—250 W per phase) at 60 Hz

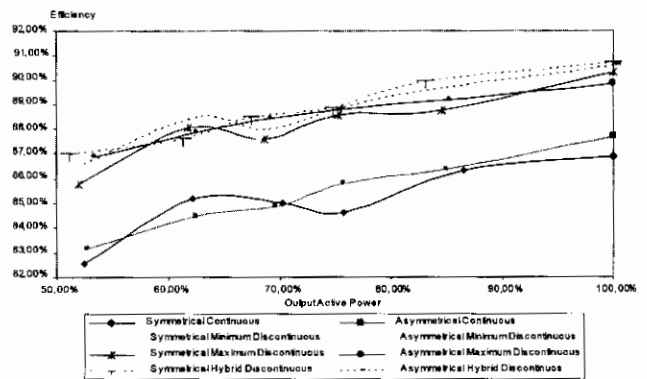


Fig. 18. Experimentally obtained efficiency for the inverter with a two phase load (RL type) versus the variation of the output power (normalized as a function of the rated value—36 W per phase) at 10 Hz

point to be made is that the machine itself saturates when the currents are close to their rated value.

All the modulations studied in this paper were implemented and efficiency tests were performed using a 250 W inductive load per phase. Fig. 17 presents the results for operation at the nominal frequency while Fig. 18 shows the results at 10 Hz. As observed, the discontinuous modulations presented the best performance, mainly at low frequencies.

The discontinuous vector modulation proposed is the best solution among the analyzed modulations. Besides reducing the number of commutations, the distribution of conduction losses is better in this case.

Due to the non regular hexagon, the number of commutations for each leg is not the same in discontinuous vector modulation. In this manner, the common leg was chosen to present the least number of commutations since it conducts the highest current in amplitude.

## 5. Conclusions

An important point to be made is that the two-phase motor, when compared to the PSC motor, presents an improved torque characteristic and uses the voltage available at the DC bus in a more efficient manner. The three-phase inverter is used in order to obtain the phase voltages and due to the fact that a capacitive divider, which presents certain inconveniences at low frequencies (voltage unbalance), is not necessary.

Space vector modulation is used since it allows greater design freedom; so much, in fact, that four modulation (symmetrical and asymmetrical) types are presented. Hybrid discontinuous space vector modulation was chosen. This modulation technique was chosen due to its small number of switching cycles and since it allows better current distribution among the legs. Another important detail is that the common leg conducts a larger load current and, therefore, is subject to greater losses, thus, the choice of hybrid discontinuous space vector modulation.

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**Denizar Cruz Martins** (Non-member) was born in Sao Paulo, SP, Brazil, on April 24, 1955. He received the B.S. and M.S. degrees in electrical engineering from the Federal University of Santa Catarina, Florianópolis, SC, Brazil, in 1978 and 1981, respectively, and the Ph.D. degree in electrical engineering from the Polytechnic National Institute of Toulouse, Toulouse, France, in 1986. He is presently Titular Professor in the Department of Electrical Engineering at Federal University of Santa Catarina, Brazil.



**Luiz Candido Tomaselli** (Non-member) was born in Ibirama, Brazil, on March 23, 1975. He received a Ph.D. degree in electrical engineering from Federal University from Santa Catarina in 2004. He has worked on analysis of single and two phase induction motors, vector modulation, and digital control of converters. Member of the Brazilian Power Electronics Society.



**Telles Brunelli Lazzarin** (Non-member) was born in Criciúma, Santa Catarina State, Brazil, on August 16, 1979. He received a B.Sc. degree in electrical engineering from the Federal University of Santa Catarina, Florianópolis, Brazil, in 2004. He is presently a M.S. student in electrical engineering at the same university.



**Ivo Barbi** (Non-member) was born in Gaspar, Santa Catarina, Brazil in 1949. He received the B.S. and M.S. degrees in electrical engineering from the Federal University of Santa Catarina, Florianópolis, Brazil, in 1973 and 1976, respectively, and the Ph.D. degree from the Polytechnic National Institute of Toulouse, Toulouse, France, in 1979. He founded the Brazilian Power Electronics Society, the Power Electronics Institute of the Federal University of Santa Catarina, and created the Brazilian Power Electronics Conference. Currently, he is Professor of the Power Electronics Institute, Federal University of Santa Catarina.

