CAPÍTULO

9

MODELOS DE ESTADO PARA O MOTOR DE INDUÇÃO

9.1 VARIÁVEIS DQ

Seja as equações elétricas do motor de indução, escritas sob a forma de variáveis dq, para um referencial genérico, representados pela expressão (9.1).

$$\begin{bmatrix} v_{S_d} \\ v_{S_q} \\ \hline v_{R_d} \\ v_{R_q} \end{bmatrix} = \left[\begin{array}{cc|cc} R_S + p\mathbb{L}_S & -\mathbb{L}_S \dot{\Psi} n & pm_{SR} & -m_{SR}\dot{\Psi} n \\ \mathbb{L}_S \dot{\Psi} n & R_S + p\mathbb{L}_S & m_{SR}\dot{\Psi} n & pm_{SR} \\ \hline pm_{SR} & -m_{SR}\left(\dot{\Psi}-\dot{\theta}\right)n & R_R + p\mathbb{L}_R & -n\left(\dot{\Psi}-\dot{\theta}\right)\mathbb{L}_R \\ m_{SR}\left(\dot{\Psi}-\dot{\theta}\right)n & pm_{SR} & n\left(\dot{\Psi}-\dot{\theta}\right)\mathbb{L}_R & R_R + p\mathbb{L}_R \end{array}\right] \begin{bmatrix} i_{S_d} \\ i_{S_q} \\ \hline i_{R_d} \\ i_{R_q} \end{bmatrix} \qquad (9.1)$$

As equações (9.1) podem ser reescritas segundo as expressões (9.2).

$$\begin{bmatrix} v_{S_d} \\ v_{S_q} \\ \hline v_{R_d} \\ v_{R_q} \end{bmatrix} = p\left[\begin{array}{cc|cc} \mathbb{L}_S & 0 & m_{SR} & 0 \\ 0 & \mathbb{L}_S & 0 & m_{SR} \\ \hline m_{SR} & 0 & \mathbb{L}_R & 0 \\ 0 & m_{SR} & 0 & \mathbb{L}_R \end{array}\right] \begin{bmatrix} i_{S_d} \\ i_{S_q} \\ \hline i_{R_d} \\ i_{R_q} \end{bmatrix} +$$

$$+ \left[\begin{array}{cc|cc} R_S & -\mathbb{L}_S \dot{\Psi} n & 0 & -m_{SR}\dot{\Psi} n \\ \mathbb{L}_S \dot{\Psi} n & R_S & m_{SR}\dot{\Psi} n & 0 \\ \hline 0 & -m_{SR}\left(\dot{\Psi}-\dot{\theta}\right)n & R_R & -n\left(\dot{\Psi}-\dot{\theta}\right)\mathbb{L}_R \\ m_{SR}\left(\dot{\Psi}-\dot{\theta}\right)n & 0 & n\left(\dot{\Psi}-\dot{\theta}\right)\mathbb{L}_R & R_R \end{array}\right] \begin{bmatrix} i_{S_d} \\ i_{S_q} \\ \hline i_{R_d} \\ i_{R_q} \end{bmatrix} \qquad (9.2)$$

Assim:

$$\mathbf{v} = p\mathbf{Z}_4\mathbf{i} + \mathbf{Z}_3\mathbf{i} \qquad (9.3)$$

$$p\mathbf{Z_4 i} = -\mathbf{Z_3 i} + \mathbf{v} \tag{9.4}$$

$$p\mathbf{i} = -\mathbf{Z_4}^{-1}\mathbf{Z_3 i} + \mathbf{Z_4}^{-1}\mathbf{v} \tag{9.5}$$

onde:

$$\mathbf{Z_4} = \left[\begin{array}{cc|cc} \mathbb{L}_S & 0 & m_{SR} & 0 \\ 0 & \mathbb{L}_S & 0 & m_{SR} \\ \hline m_{SR} & 0 & \mathbb{L}_R & 0 \\ 0 & m_{SR} & 0 & \mathbb{L}_R \end{array}\right] \tag{9.6}$$

$$\mathbf{Z_4}^{-1} = \frac{1}{\mathbb{L}_S\mathbb{L}_R - m_{SR}^2}\left[\begin{array}{cc|cc} \mathbb{L}_R & 0 & -m_{SR} & 0 \\ 0 & \mathbb{L}_R & 0 & -m_{SR} \\ \hline -m_{SR} & 0 & \mathbb{L}_S & 0 \\ 0 & -m_{SR} & 0 & \mathbb{L}_S \end{array}\right] \tag{9.7}$$

$$\mathbf{Z_4}^{-1}\mathbf{Z_3} = \frac{1}{\sigma}\left[\begin{array}{cc|cc} \mathbb{L}_R & 0 & -m_{SR} & 0 \\ 0 & \mathbb{L}_R & 0 & -m_{SR} \\ \hline -m_{SR} & 0 & \mathbb{L}_S & 0 \\ 0 & -m_{SR} & 0 & \mathbb{L}_S \end{array}\right]\left[\begin{array}{cc|cc} R_S & -\mathbb{L}_S\dot{\Psi}n & 0 & -m_{SR}\dot{\Psi}n \\ \mathbb{L}_S\dot{\Psi}n & R_S & m_{SR}\dot{\Psi}n & 0 \\ \hline 0 & -m_{SR}\left(\dot{\Psi}-\dot{\theta}\right)n & R_R & -n\left(\dot{\Psi}-\dot{\theta}\right)\mathbb{L}_R \\ m_{SR}\left(\dot{\Psi}-\dot{\theta}\right)n & 0 & n\left(\dot{\Psi}-\dot{\theta}\right)\mathbb{L}_R & R_R \end{array}\right] \tag{9.8}$$

onde:

$$\frac{1}{\sigma} = \frac{1}{\mathbb{L}_R\mathbb{L}_S - m_{SR}^2} \tag{9.9}$$

Substituindo-se as expressões (9.7), (9.8) e (9.9) na expressão (9.5), encontra-se a expressão (9.10).

$$p\begin{bmatrix} i_{S_d} \\ i_{S_q} \\ \hline i_{R_d} \\ i_{R_q} \end{bmatrix} = \frac{1}{\sigma}\begin{bmatrix} -R_S\mathbb{L}_R & \mathbb{L}_S\mathbb{L}_R\dot{\Psi}n + \\ & -m_{SR}^2\left(\dot{\Psi}-\dot{\theta}\right)n & m_{SR}R_R & m_{SR}\mathbb{L}_R\dot{\theta}n \\ -\mathbb{L}_S\mathbb{L}_R\dot{\Psi}n + & & & \\ +m_{SR}^2\left(\dot{\Psi}-\dot{\theta}\right)n & -R_S\mathbb{L}_R & -m_{SR}\mathbb{L}_R\dot{\theta}n & m_{SR}R_R \\ \hline m_{SR}R_S & -m_{SR}\mathbb{L}_S\dot{\theta}n & -R_R\mathbb{L}_S & \mathbb{L}_S\mathbb{L}_R\left(\dot{\Psi}-\dot{\theta}\right)n + \\ & & & -m_{SR}^2\dot{\Psi}n \\ m_{SR}\mathbb{L}_S\dot{\theta}n & m_{SR}R_S & -\mathbb{L}_S\mathbb{L}_R\left(\dot{\Psi}-\dot{\theta}\right)n + & -R_R\mathbb{L}_S \\ & & +m_{SR}^2\dot{\Psi}n & \end{bmatrix}\begin{bmatrix} i_{S_d} \\ i_{S_q} \\ \hline i_{R_d} \\ i_{R_q} \end{bmatrix} +$$

$$+\frac{1}{\sigma}\begin{bmatrix} \mathbb{L}_R & 0 & -m_{SR} & 0 \\ 0 & \mathbb{L}_R & 0 & -m_{SR} \\ \hline -m_{SR} & 0 & \mathbb{L}_S & 0 \\ 0 & -m_{SR} & 0 & \mathbb{L}_S \end{bmatrix}\begin{bmatrix} v_{S_d} \\ v_{S_q} \\ \hline v_{R_d} \\ v_{R_q} \end{bmatrix} \qquad (9.10)$$

As expressões (9.10) representam as equações elétricas da máquina sob a forma de estado. Caso se deseje o referencial no estator, basta fazer $\dot{\Psi} = 0$. No modelo está incluído o número de pares de pólos n. Para o rotor em curto-circuito, toma-se $v_{R_d} = v_{R_q} = 0$.

Para a equação mecânica tem-se:

$$Te = Jp\dot{\theta} + D\dot{\theta} + T_L \qquad (9.11)$$

Assim:

$$p\dot{\theta} = \frac{Te}{J} - \frac{D}{J}\dot{\theta} - \frac{T_L}{J} \qquad (9.12)$$

mas,

$$Te = nm_{SR}\left(i_{S_q}i_{R_d} - i_{S_d}i_{R_q}\right) \qquad (9.13)$$

Portanto:

$$p\dot{\theta} = \frac{nm_{SR}}{J}\left(i_{S_q} i_{R_d} - i_{S_d} i_{R_q}\right) - \frac{D}{J}\dot{\theta} - \frac{T_L}{J} \tag{9.14}$$

9.2 VARIÁVEIS COMPONENTES SIMÉTRICAS INSTANTÂNEAS

Seja as equações elétricas do motor, escritas sob a forma de componentes simétricas instantâneas para um referencial genérico, representada pela expressão (9.15).

$$\begin{bmatrix} v_{S_+} \\ v_{R_+} \end{bmatrix} = \begin{bmatrix} R_S + \mathbb{L}_S\left(p + jn\dot{\Psi}\right) & m_{SR}\left(p + jn\dot{\Psi}\right) \\ m_{SR}\left(p + jn\dot{\Psi} - jn\dot{\theta}\right) & R_R + \mathbb{L}_R\left(p + jn\dot{\Psi} - jn\dot{\theta}\right) \end{bmatrix} \begin{bmatrix} i_{S_+} \\ i_{R_+} \end{bmatrix} \tag{9.15}$$

A expressão (9.15) é reescrita e passa a ser representada pela expressão (9.16).

$$\begin{bmatrix} v_{S_+} \\ v_{R_+} \end{bmatrix} = p\begin{bmatrix} \mathbb{L}_S & m_{SR} \\ m_{SR} & \mathbb{L}_R \end{bmatrix} \begin{bmatrix} i_{S_+} \\ i_{R_+} \end{bmatrix} + \begin{bmatrix} R_S + jn\dot{\Psi}\mathbb{L}_S & jn\dot{\Psi} m_{SR} \\ jn\left(\dot{\Psi} - \dot{\theta}\right) m_{SR} & R_R + jn\left(\dot{\Psi} - \dot{\theta}\right)\mathbb{L}_R \end{bmatrix} \begin{bmatrix} i_{S_+} \\ i_{R_+} \end{bmatrix} \tag{9.16}$$

Assim:

$$\mathbf{v} = p\mathbf{Z_4 i} + \mathbf{Z_3 i} \tag{9.17}$$

$$p\mathbf{i} = -\mathbf{Z_4}^{-1}\mathbf{Z_3 i} + \mathbf{Z_4}^{-1}\mathbf{v} \tag{9.18}$$

Onde:

$$\mathbf{Z_4} = \begin{bmatrix} \mathbb{L}_S & m_{SR} \\ m_{SR} & \mathbb{L}_R \end{bmatrix} \tag{9.19}$$

$$\mathbf{Z_3} = \left[\begin{array}{c|c} R_S + jn\dot{\Psi}\mathbb{L}_S & jn\dot{\Psi} m_{SR} \\ \hline jn\left(\dot{\Psi} - \dot{\theta}\right) m_{SR} & R_R + jn\left(\dot{\Psi} - \dot{\theta}\right)\mathbb{L}_R \end{array} \right] \tag{9.20}$$

$$\mathbf{Z_4}^{-1} = \frac{1}{\mathbb{L}_S\mathbb{L}_R - m_{SR}^{\;2}} \begin{bmatrix} \mathbb{L}_R & -m_{SR} \\ -m_{SR} & \mathbb{L}_S \end{bmatrix} \tag{9.21}$$

Assim:

$$p\mathbf{i} = -\frac{1}{\sigma}\begin{bmatrix} \mathbb{L}_R & -m_{SR} \\ -m_{SR} & \mathbb{L}_S \end{bmatrix} \left[\begin{array}{c|c} R_S + jn\dot{\Psi}\mathbb{L}_S & jn\dot{\Psi} m_{SR} \\ \hline jn\left(\dot{\Psi} - \dot{\theta}\right) m_{SR} & R_R + jn\left(\dot{\Psi} - \dot{\theta}\right)\mathbb{L}_R \end{array} \right] \left[\frac{i_{S_+}}{i_{R_+}} \right] + \frac{1}{\sigma}\begin{bmatrix} \mathbb{L}_R & -m_{SR} \\ -m_{SR} & \mathbb{L}_S \end{bmatrix} \left[\frac{v_{S_+}}{v_{R_+}} \right] \tag{9.22}$$

$$p\mathbf{i} = -\frac{1}{\sigma}\left[\begin{array}{c|c} \begin{array}{c} \mathbb{L}_R\left(R_S + jn\dot{\Psi}\mathbb{L}_S\right) + \\ -m_{SR}^{\;2} jn\left(\dot{\Psi} - \dot{\theta}\right) \end{array} & \begin{array}{c} j\mathbb{L}_R n\dot{\Psi} m_{SR} + \\ -m_{SR}\left(R_R + jn\left(\dot{\Psi} - \dot{\theta}\right)\mathbb{L}_R\right) \end{array} \\ \hline \begin{array}{c} -m_{SR}\left(R_S + jn\dot{\Psi}\mathbb{L}_S\right) + \\ +jn\left(\dot{\Psi} - \dot{\theta}\right) m_{SR}\mathbb{L}_S \end{array} & \begin{array}{c} -jn\dot{\Psi} m_{SR}^{\;2} + \\ +\mathbb{L}_S\left(R_R + jn\left(\dot{\Psi} - \dot{\theta}\right)\mathbb{L}_R\right) \end{array} \end{array} \right] \left[\frac{i_{S_+}}{i_{R_+}} \right] + \frac{1}{\sigma}\begin{bmatrix} \mathbb{L}_R & -m_{SR} \\ -m_{SR} & \mathbb{L}_S \end{bmatrix} \left[\frac{v_{S_+}}{v_{R_+}} \right] \tag{9.23}$$

onde:

$$\frac{1}{\sigma} = \frac{1}{\mathbb{L}_R\mathbb{L}_S - m_{SR}^{\;2}} \tag{9.24}$$

Portanto:

$$p\mathbf{i} = \mathbf{Ai} + \mathbf{Bv} \tag{9.25}$$

$$\mathbf{A} = \frac{1}{\sigma}\left[\begin{array}{c|c} -R_S\mathbb{L}_R - jn\dot{\Psi}\mathbb{L}_R\mathbb{L}_S + jn\left(\dot{\Psi} - \dot{\theta}\right) m_{SR}^{\;2} & R_R m_{SR} - jn\dot{\theta} m_{SR}\mathbb{L}_R \\ \hline R_S m_{SR} + jn\dot{\theta} m_{SR}\mathbb{L}_S & jn\dot{\Psi} m_{SR}^{\;2} - R_R\mathbb{L}_S - jn\left(\dot{\Psi} - \dot{\theta}\right)\mathbb{L}_S\mathbb{L}_R \end{array} \right] \tag{9.26}$$

$$\mathbf{B} = \frac{1}{\sigma}\begin{bmatrix} \mathbb{L}_R & -m_{SR} \\ -m_{SR} & \mathbb{L}_S \end{bmatrix} \tag{9.27}$$

Em seguida será tratada a equação mecânica.

$$Te = 2nm_{SR}\mathbb{I}\left(i_{S_+} i_{R_-}\right) \tag{9.28}$$

Assim:

$$Te = Jp\dot{\theta} + D\dot{\theta} + T_L \tag{9.29}$$

$$p\dot{\theta} = \frac{Te}{J} - \frac{D}{J}\dot{\theta} - \frac{T_L}{J} \tag{9.30}$$

Caso se deseje o referencial colocado no estator, basta fazer $\dot{\Psi} = 0$.